Technical Notes

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Turbulent Boundary Layers as Affected by Weak Magnetohydrodynamic Forces

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OVER the past several years a collaborative effort has been underway at the University of Rhode Island to develop a new integral technique for analyzing the turbulent boundary layer. This work has resulted in seven publications which are documented in Ref. 1. The approach is extended here to include the effects of weak magnetohydrodynamic forces.

The equation of motion for steady two-dimensional turbulent boundary-layer flow in the presence of a uniform transverse magnetic field for the case of small magnetic Reynolds number are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x}-\frac{\sigma B_o^2}{\rho}u+\frac{1}{\rho}\frac{\partial \tau}{\partial y} \tag{2}$$

where u and v are the velocity components parallel and normal to the surface, B_o is the applied magnetic flux density, and τ is the total shear stress. A freestream force balance implies the pressure gradient is given by

$$U\frac{\mathrm{d}U}{\mathrm{d}x}=-\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x}-\frac{\sigma B_o^2}{\rho}U \tag{3}$$

The approach to be used in this analysis is to nondimensionalize the momentum equation using law-of-the-wall variables, and integrate across the boundary layer with respect to y^+. This requires a suitable velocity profile assumption.

Napolitano[2] concludes that the two-layer model for the turbulent boundary layer is still valid in the presence of magnetic fields for sufficiently small magnetic Reynolds numbers. Therefore, in the inner quasi-equilibrium layer, the shear stress may be written as

$$\tau=\rho K^2 y^2 \left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

Bocheninskii and Tananaev[3] have complied velocity profile data for channel flows in the presence of magnetic fields. Their data show that the slope of the overlap layer is not changed by the fields, implying that the constant in Eq. (4) may be taken as von Karman's constant ($K=0.4$).

Received Jan. 7, 1976; revision received April 12, 1976.

Index category: Boundary Layers and Convective Heat Transfer – Turbulent; Plasma Dynamics and MHD.

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Near-the-wall convective accelerations are small, and the boundary layer can be approximated by

$$0=U\left(\frac{\mathrm{d}U}{\mathrm{d}x}+\frac{\sigma B_o^2}{\rho}\right)-\frac{\sigma B_o^2}{\rho}u+\frac{1}{\rho}\frac{\partial \tau}{\partial y} \tag{5}$$

Integrating this equation from the wall outward with respect to y^+ gives

$$\tau=\rho u_*^2+\nu\sigma B_o^2\psi^+-\frac{\mu U}{\mu_*}\left(\frac{\mathrm{d}U}{\mathrm{d}x}+\frac{\sigma B_o^2}{\rho}\right)y^+ \tag{6}$$

where u_* is the shear velocity and

$$\psi^+=\int_0^{y^+} u^+\,\mathrm{d}y^+ \tag{7}$$

is a nondimensional steam function. Equating Eqs. (4) and (6), and solving for the velocity gradient results in

$$\frac{\partial u^+}{\partial y^+}=\frac{(\alpha\psi^+-\beta y^++1)^{1/2}}{Ky^+} \tag{8}$$

where

$$\alpha=\nu\sigma B_o^2/\rho u_*^2 \text{ and } \beta=\left(\frac{\mathrm{d}U}{\mathrm{d}x}+\frac{\sigma B_o^2}{\rho}\right)\nu U/u_*^3$$

are the magnetic force and pressure gradient parameters, respectively. This equation defines the required law-of-the-wall velocity profile.

To integrate Eq. (8) an initial condition is needed. The data of Bocheninskii and Tananaev[3] show a slight but rather insignificant decrease in the thickness of the viscous sublayer (2-3%). This change of sublayer thickness is well within the existing error limits for turbulent velocity profile data. Therefore, for the calculations presented in this paper the sublayer thickness is assumed unchanged.

Equations (1-3) may be solved for the local skin friction by defining a stream function, ψ and eliminating the velocity v from Eq. (2) in terms of ψ by use of Eq. (1). This gives

$$u_*u^+\frac{\partial}{\partial x}(u_*u^+)-u_*\frac{\partial\psi^+}{\partial x}\frac{\partial}{\partial y^+}(u_*u^+)=-\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x}$$
$$-\frac{\sigma B_o^2}{\rho}u_*u^++\frac{u_*}{\mu}\frac{\partial\tau}{\partial y^+} \tag{9}$$

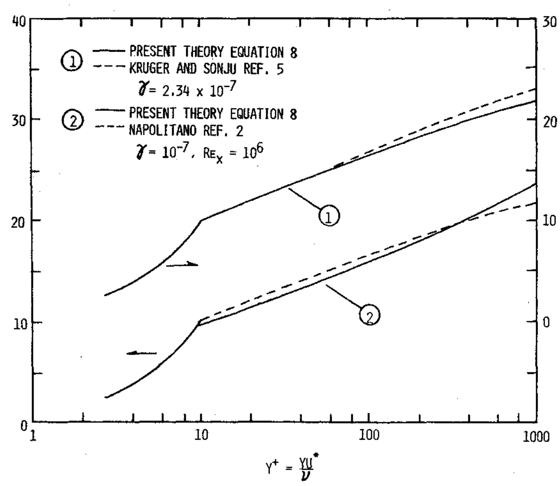

Fig. 1 Comparison of law-of-the-wall velocity profiles.

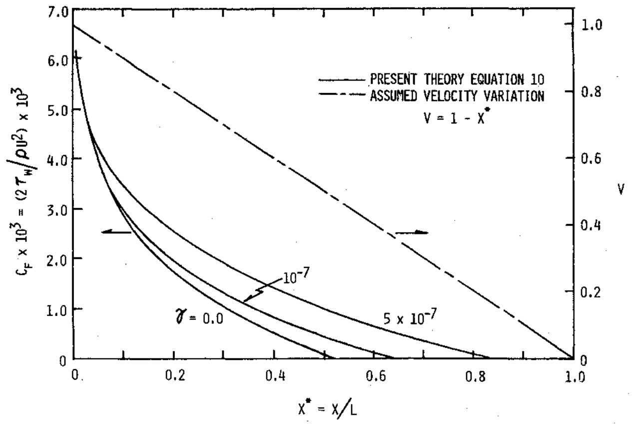

Fig. 2 Skin friction results – adverse pressure gradient flow.

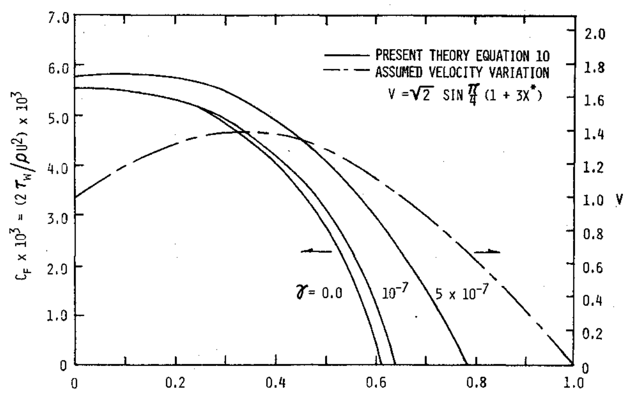

Fig. 3 Skin friction results – a favorable followed by strongly adverse pressure gradient flow.

The dependent variables u^+ and ψ^+ are both functions of y^+, α, and β which are functions of x. After calculating all streamwise gradients by use of the chain rule for partial derivatives, Eq. (9) may be integrated across the boundary layer. After some algebraic manipulations this yields

$$\frac{d\lambda}{dx^*} = RV(1-H)/G + V'\lambda/V \tag{10}$$

in which the following quantities have been defined for convenience: $x^* = x/L$, $V = U/U_o$, $R = U_oL/\nu$, and $\lambda = V(2/C_f)^{1/2}$ where U_o and L are constant reference quantities.

Equation (10) is a first-order ordinary differential equation for skin friction as a function of x^*. The quantities G and H are integral coefficients defined by quadratures over the assumed velocity profile, Eq. (8). This equation contains a built-in separation criteria, in that if G goes to zero, then $d\lambda/dx^*$ goes to infinity, and C_f goes to zero. All that is needed to solve Eq. (10) is an initial value of C_f and knowledge of the outer flow variations. For further details of this derivation see Ref. 4.

Two previous studies similar to the present analysis were carried out by Napolitano,[2] and by Kruger and Sonju.[5] The former considered an isobaric freestream, while the latter considered a constant freestream velocity. The results obtained by application of Eq. (10) to those problems compare qualitatively with the earlier results, being approximately 15% lower in the first instance and 20% higher in the second. Figure 1 shows a comparison of the velocity profiles used in these early studies with Eq. (8).

Since the present analysis includes pressure gradient effects, several sample calculations for different pressure gradient flows, at different values of the interaction parameter $\gamma = \sigma B_o^2 \nu / \rho U_o^2$, are presented on Figs. 2-4. Figure 2 shows the results for an adverse pressure gradient flow, Fig. 3 is for a flow which initially experiences a favorable pressure gradient before becoming adverse, and Fig. 4 presents results for a relaxing flow. These plots show skin friction as a function of x^*, and also show the assumed variation of the freestream velocity. In all cases the skin friction is higher with the magnetic fields present than for the zero interaction case. In the adverse pressure gradient cases the presence of the magnetic fields are assumed to be applied only near the separation point, rather than over the entire surface.

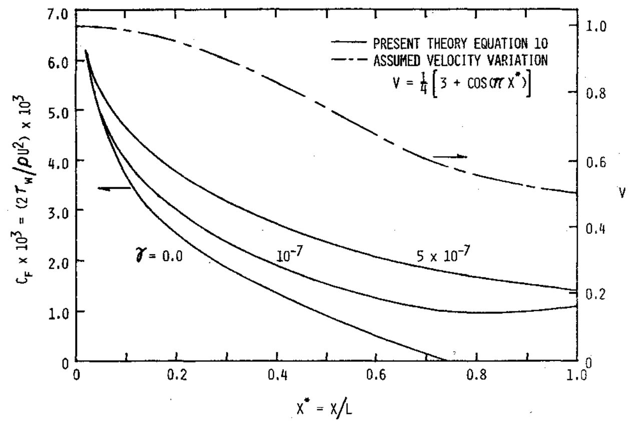

Fig. 4 Skin friction results – relaxing flow.

For the results presented in this paper, the interaction parameter γ and the length Reynolds number R are of the order of magnitude 10^{-7} and 10^7, respectively. The product of the interaction parameter and the Reynolds number represents the ratio of the magnetic pressure to the dynamic pressure. For the magnetic fields to have a significant effect the product γR must be of order one. Therefore, for flow situations where the length Reynolds number becomes very large, only weak magnetic fields are necessary to alter the pressure distribution and significantly change the skin friction.

References

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Simplified Modeling of Multiquantum Deactivation Reactions

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Introduction

COMPUTER modeling of pulsed and cw chemical lasers generally involves a large number of reactions,[1-3] since

Received Feb. 9, 1976.

Index categories: Reactive Flows; Lasers; Thermochemistry and Chemical Kinetics.

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